Bond return predictability: Macro factors and machine learning methods

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Main Objectives

- **Key Questions:**
  - How macroeconomic variables help **predict bond risk premia**?
  - How the **nonlinear relationship** between macroeconomic variables and bond returns can be modelled?

- **This Paper**
  - Focus on **Chinese Bonds**
  - **Much larger set of macroeconomic variables**
  - Gain of macroeconomic variables in comparison to forward rates

1. Introduction

- **Bond returns**
  - Cochrane and Piazzesi (2005): linear combination of **forward rates** can explain a substantial portion of treasury bond return variation
  - Most recent literature: **macroeconomic variables** as important determinants of the yield curve (Gargano et al; Joslin et al; Ludvigson & Ng; **Fernandes e Nunes 2021**)

- **OLS x ML**
  - ML outperform OLS in **variable selection** and dimensionality reduction
  - Prior studies show that the association between explanatory variables and bond returns is **nonlinear**

1. Introduction

- **Macroeconomic Impact: Channels**
  - **Brandt and Wang (2003):** Excess returns are explained by the unexpected inflation rate, increasing investors risk aversion and raising bond risk premia
  - **Wachter (2006)**: difference between investors borrowing behavior
  - **Jermann (2013)**: infrastructure investment in a production-based model
  - **Cooper & Priestley (2009)**: Economic growth influences bond risk premia by shaping investors expectation about future economic condition
  - **Fan et al (2012):** Monetary policy affects the term structure of interest rates, potentially influencing treasury bond risk premia

1. Introduction

- **Chinese Bond Market**
  - Before 2015 interest rates on all terms were set by the PBoC (official rates)
  - Exchange, OTC + **Interbank Bond Market** with institutional investors, mostly commercial banks (**SOEs**)
    - **How effective are macroeconomic variables in this context?**

How effective are macroeconomic variables in this context?

2. Methodology

- **Bond excess returns**
  - Zero-coupon bond with a remaining maturity of **n** years in year **t** paying 1 CNY
  - **1-year log excess return:**

$$
\begin{aligned}
xr_{t+1}^{(n)} &= p_{t+1}^{(n-1)} - p_t^{(n)} - y_t^{(1)} \\
&= -(n-1)\left(y_{t+1}^{(n-1)} - y_t^{(n)}\right) + \left(y_t^{(n)} - y_t^{(1)}\right),
\end{aligned}
\qquad (1)
$$

- $p_t^{(n)}$ and $y_t^{(n)}$ are the logarithm of the price and continuously compounded yield in year t
- YTM of a 1-year bond is represented by $y_t^{(1)}$
- **Forward Rate:**

$$f_t^{(n)} = ny_t^{(n)} - (n-1)y_t^{(n-1)}$$

2. Methodology

- **Return predictability**

$$
\begin{aligned}
xr_{t+1}^{(n)} &= p_{t+1}^{(n-1)} - p_t^{(n)} - y_t^{(1)} \\
&= -(n-1)\boxed{\left(y_{t+1}^{(n-1)} - y_t^{(n)}\right)} + \boxed{\left(y_t^{(n)} - y_t^{(1)}\right),}
\end{aligned}
\qquad (1)
$$

Variation of **Yield**

Variation of **Slope**

- Any variables capable of predicting the variations in the bond yield or the slope of the yield curve are expected to **help predict future bond excess returns**

2. Methodology

- **Information set**
  - Information set for investors in year **t** is a multidimensional state vector, π_t , and the relationship between the variables and the 1-year holding period bond excess return can be written as:

$$xr_{t+1}^{(n)} = g(\pi_t; N) + \varepsilon_t, \tag{2}$$

- N: vector of bonds with different maturities

2. Forecasting methods: OLS

- **OLS**
  - **1 year yield and 1-year forward rates** with maturities ranging from 2 to 10 years as predictors

$$xr_{t+1}^{(n)} = \nu + a_1 y_t^{(1)} + a_2 f_t^{(2)} + \cdots + a_s f_t^{(n)} + \zeta_t, \quad n = 2, \ldots, 10,$$

  - **(+) Set of macroeconomic factors**

$$xr_{t+1}^{(n)} = \chi + a_1 y_t^{(1)} + a_2 f_t^{(2)} + \cdots + a_s f_t^{(n)} + c_1 o_{1,t} + \cdots + c_m o_{m,t} + \xi_t,$$

  - **Limits the number of macroeconomic factors to 15**

2. Forecasting methods: Machine Learning

- **Linear**
  - **Principal Component Regression (PCR)**
    - Transforms the set of 102 macroeconomic variables into a set **of principal components (PCs)**
  - **Partial Least Squares (PLS)**
  - **Penalized regressions: Ridge, Lasso and Enet**
- **Non-Linear**
  - **Regression Trees / Random Forest**
  - **Neural Networks**

- Rolling training window of 8 years (Train/Valid split of 85%/15%)

2. Performance evaluation

- **Statistical performance measure**

$$R_{oos}^2 = 1 - \frac{\sum_{\tau=1}^{T-1}\left(xr_{\tau+1}^{(n)} - \hat{x}r_{\tau+1}^{(n)}(Predictive)\right)^2}{\sum_{\tau=1}^{T-1}\left(xr_{\tau+1}^{(n)} - \overline{xr}_{\tau+1}^{(n)}\right)^2},$$

  - A **positive** value indicates that the model under **investigation outperforms the historical average benchmark.**

2. Performance evaluation

- **Economic performance measure**
  - **Two asset-allocation exercises**
    - **Univariate** case: The investor selects bonds from the 2-, 3-, 5-, 7- and 10-year maturities with the 1y bond yield being the RF rate.
    - **Multivariate** case: The investor considers risky bonds across the same 5 maturities
  - A mean-variance utility investor allocates across **J** risky bonds with n-year maturities with this utility:

Risk free rate

$$U_t(r_t) = E_t(w_t r_t + r_{t,f}) - \frac{1}{2}\gamma \mathrm{Var}_t(w_t r_t + r_{t,f}),$$

Risk-aversion coefficient

Excess return of the risky asset

Weight of the risky asset

2. Performance evaluation

- **Economic performance measure**

Risk free rate

$$U_t(r_t) = E_t(w_t r_t + r_{t,f}) - \frac{1}{2}\gamma \mathrm{Var}_t(w_t r_t + r_{t,f}),$$

Risk-aversion coefficient

Excess return of the risky asset

Weight of the risky asset

- Considering the excess returns already defined, the return on an investment portfolio at time t+1 is defined as follows:

$$R_{t+1} = 1 + y_t^{(1)} + w_t' x r_{t+1}^{(n)},$$

- And the realized utility at year t+1 as follows:

$$u_{t+1} = R_{t+1} - \frac{\gamma}{2(1+\gamma)} R_{t+1}.$$

2. Performance evaluation

- **Economic performance measure**
  - The Certainty Equivalent Return (CER) is obtained by equating the average utility of the predictive method with the average utility of any of the benchmark models, such as the EH model.
  - **But there's still the issue that these economic gains aren't significantly greater than zero:**

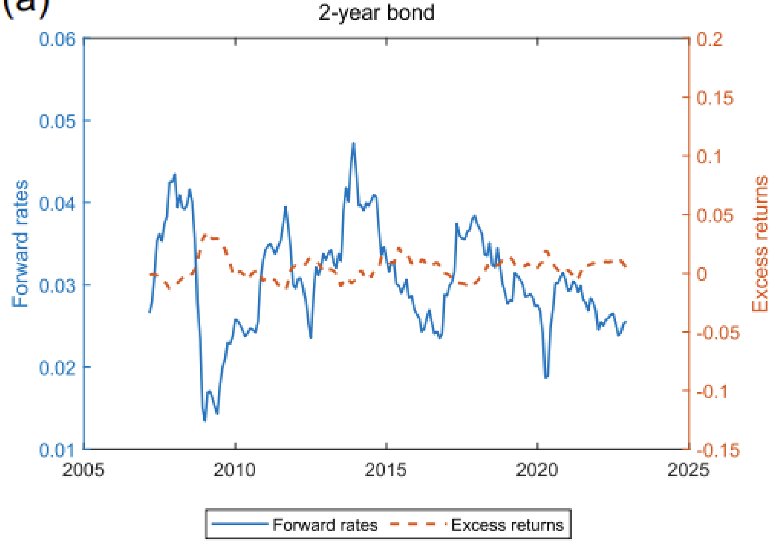
$$u_{t+1,Predictive} - u_{t+1,Benchmark} = \upsilon^{(n)} + \delta_{t+1}, \qquad (17)$$

- **A pairwise test via this regression to ensure that the economic gains are significantly greater in the predictive model.**

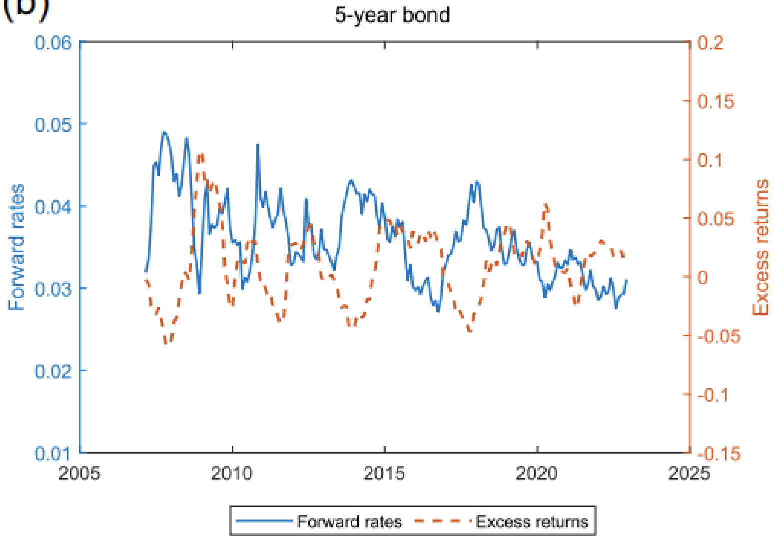
3. Data

- **Monthly** data on the Chinese zero-coupon treasury bond yields from **1 to 10 years**
- **102 macroeconomic variables**
  - macro-prosperity index, output, consumption and retail, price indexes, interest rates, money and credit, investment, real estate, tax, trade, FX, stocks, monetary policy
- Sample from **March 2006 to December 2022**

(a)



(b)



4. Forecasting performance evaluation: forward rates

	$xr^{(2)}_{t+1}$ (%)	$xr^{(3)}_{t+1}$ (%)	$xr^{(4)}_{t+1}$ (%)	$xr^{(5)}_{t+1}$ (%)	$xr^{(6)}_{t+1}$ (%)	$xr^{(7)}_{t+1}$ (%)	$xr^{(8)}_{t+1}$ (%)	$xr^{(9)}_{t+1}$ (%)	$xr^{(10)}_{t+1}$ (%)
Panel A: OLS									
OLS	26***	19***	10**	2**	1**	−9	−13	−15	−22
	(0.00)	(0.00)	(0.01)	(0.01)	(0.02)				
Panel B: Linear machine learning methods									
PCR and PLS									
PCR (3 components)	19**	6**	−7	−25	−22	−36	−30	−29	−34
	(0.01)	(0.02)							
PCR (5 components)	34**	34***	28**	21**	20**	13**	11**	7**	−0.3
	(0.01)	(0.00)	(0.00)	(0.00)	(0.01)	(0.01)	(0.01)	(0.02)	
PLS (5 components)	36***	30***	26***	18**	18**	6**	5**	0.2**	−8
	(0.00)	(0.00)	(0.00)	(0.01)	(0.01)	(0.02)	(0.02)	(0.02)	
PLS (8 components)	36***	29***	24**	18**	20**	8**	7**	4**	−3
	(0.00)	(0.00)	(0.01)	(0.01)	(0.01)	(0.01)	(0.02)	(0.02)	
Penalized linear regressions									
Ridge	36***	30***	25**	17**	19**	6**	5**	2**	−5
	(0.00)	(0.00)	(0.01)	(0.01)	(0.01)	(0.02)	(0.02)	(0.02)	

- **OLS**: Positive R2 (better than historical averages) but its performance deteriorates for long-term bonds.
- Similar pattern across PCR w/ 5+ components, PLS and Penalized Regressions

4. Forecasting performance evaluation: forward rates

	$xr^{(2)}_{t+1}$ (%)	$xr^{(3)}_{t+1}$ (%)	$xr^{(4)}_{t+1}$ (%)	$xr^{(5)}_{t+1}$ (%)	$xr^{(6)}_{t+1}$ (%)	$xr^{(7)}_{t+1}$ (%)	$xr^{(8)}_{t+1}$ (%)	$xr^{(9)}_{t+1}$ (%)	$xr^{(10)}_{t+1}$ (%)
Lasso	27**	35***	32**	16**	14**	9**	10**	1**	0.3**
	(0.01)	(0.00)	(0.01)	(0.01)	(0.01)	(0.02)	(0.02)	(0.02)	(0.03)
ENet	31**	31**	30**	21**	11**	3**	3**	3**	2**
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.02)	(0.02)	(0.02)	(0.03)
Panel C: Nonlinear machine learning methods									
Regression trees									
GBRT	26**	14**	11**	2*	3*	8**	9*	9**	5*
	(0.02)	(0.04)	(0.04)	(0.05)	(0.06)	(0.04)	(0.05)	(0.04)	(0.05)
Random forest	23**	13**	13**	5**	8**	12**	12**	10**	6**
	(0.03)	(0.03)	(0.01)	(0.02)	(0.01)	(0.02)	(0.01)	(0.02)	(0.02)
Neural networks									
NN-1 layer (3 nodes)	11*	10**	11***	11**	9**	12**	11**	12**	12**
	(0.09)	(0.01)	(0.00)	(0.00)	(0.02)	(0.01)	(0.00)	(0.00)	(0.00)
NN-1 layer (5 nodes)	11	9**	7	11**	8*	12**	12***	11**	12**
	(0.15)	(0.02)	(0.12)	(0.00)	(0.06)	(0.01)	(0.00)	(0.01)	(0.01)
NN-1 layer (7 nodes)	11	11***	11**	11**	10**	12**	11***	12**	12**
	(0.12)	(0.00)	(0.01)	(0.00)	(0.01)	(0.01)	(0.00)	(0.01)	(0.00)
NN-2 layers (3 nodes each)	9**	11***	11***	12***	10**	12**	11***	12***	12***
	(0.01)	(0.00)	(0.00)	(0.00)	(0.01)	(0.01)	(0.00)	(0.00)	(0.00)
NN-2 layers (5 nodes each)	4	9**	11***	12***	10**	11**	11***	12***	12***
	(0.25)	(0.03)	(0.00)	(0.00)	(0.01)	(0.01)	(0.00)	(0.00)	(0.00)

Surprise: **Tree/NNs do not reveal any improvements** compared with linear models.

Possible explanation: simple relationship between forward rates and short-term bond risk; little room to explore non-linearities

Conclusion: prediction capability exists but can be improved

4. Forecasting performance evaluation: fwd rates + macro

	$xr_{t+1}^{(2)}$ (%)	$xr_{t+1}^{(3)}$ (%)	$xr_{t+1}^{(4)}$ (%)	$xr_{t+1}^{(5)}$ (%)	$xr_{t+1}^{(6)}$ (%)	$xr_{t+1}^{(7)}$ (%)	$xr_{t+1}^{(8)}$ (%)	$xr_{t+1}^{(9)}$ (%)	$xr_{t+1}^{(10)}$ (%)
Panel A: OLS									
OLS	13**	−0.1	−11	−22	−24	−31	−36	−33	−41
	(0.01)								
Panel B: Linear machine learning methods									
PCR and PLS									
PCR (8 components)	−8	−3	−9	−23	−22	−32	−19	−15	−14
PCR (15 components)	−23	−21	−21	−43	−32	−49	−31	−27	−29
PLS	−61	−59	−68	−83	−83	−103	−90	−88	−93
Penalized linear regressions									
Ridge	−71	−81	−107	−144	−178	−189	−161	−169	−58
Lasso	26**	37**	30**	13**	6**	−6	−5	−5	−7
	(0.03)	(0.01)	(0.01)	(0.02)	(0.02)				
Elastic net	17**	18**	−4	−8	−25	−9	−13	−13	−14
	(0.01)	(0.01)							

OLS + LinearML
Negative performance in most maturities

4. Forecasting performance evaluation: fwd rates + macro

	$xr^{(2)}_{t+1}$ (%)	$xr^{(3)}_{t+1}$ (%)	$xr^{(4)}_{t+1}$ (%)	$xr^{(5)}_{t+1}$ (%)	$xr^{(6)}_{t+1}$ (%)	$xr^{(7)}_{t+1}$ (%)	$xr^{(8)}_{t+1}$ (%)	$xr^{(9)}_{t+1}$ (%)	$xr^{(10)}_{t+1}$ (%)
Panel C: Nonlinear machine learning methods									
Regression trees									
GBRT	31**	24**	12**	16**	20**	13**	12**	11**	10**
	(0.01)	(0.03)	(0.03)	(0.02)	(0.02)	(0.03)	(0.03)	(0.03)	(0.04)
Random forest	30**	24**	18**	12**	23**	23**	30**	30**	31**
	(0.02)	(0.02)	(0.03)	(0.04)	(0.02)	(0.02)	(0.01)	(0.01)	(0.02)
Neural networks									
Hybrid-net, 1 layer (32 nodes)	24***	50***	37***	50***	32***	52***	54***	50***	60***
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Hybrid-net, 2 layers (32, 16 nodes)	39**	57***	57***	56***	55***	55***	58***	59***	58***
	(0.01)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Hybrid-net, 3 layers (32, 16, 8 nodes)	51***	59***	61***	51***	54***	56***	59***	58***	58***
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Double-net, 1 layer each (32 nodes, 3 nodes)	10**	21**	26**	26**	26*	25*	28**	24**	25**
	(0.01)	(0.04)	(0.02)	(0.03)	(0.07)	(0.07)	(0.04)	(0.03)	(0.02)
Double-net, 2 layers, 1 layer (32, 16 nodes, 3 nodes)	12*	14*	14*	11***	11***	13***	12***	14**	15**
	(0.05)	(0.06)	(0.05)	(0.00)	(0.00)	(0.00)	(0.00)	(0.01)	(0.01)
Double-net, 3 layers, 1 layer (32, 16, 8 nodes, 3 nodes)	11**	13**	12**	13**	12***	14**	13**	13**	14**
	(0.01)	(0.01)	(0.02)	(0.01)	(0.00)	(0.01)	(0.00)	(0.00)	(0.01)
Group-ensemble-net, 1 layer each (1 node per group, 3 nodes)	11***	13**	13**	14**	12***	14**	13***	13***	14**
	(0.00)	(0.01)	(0.01)	(0.01)	(0.00)	(0.01)	(0.00)	(0.00)	(0.01)

Nonlinear ML performs much better; Hybrid-net outperforms

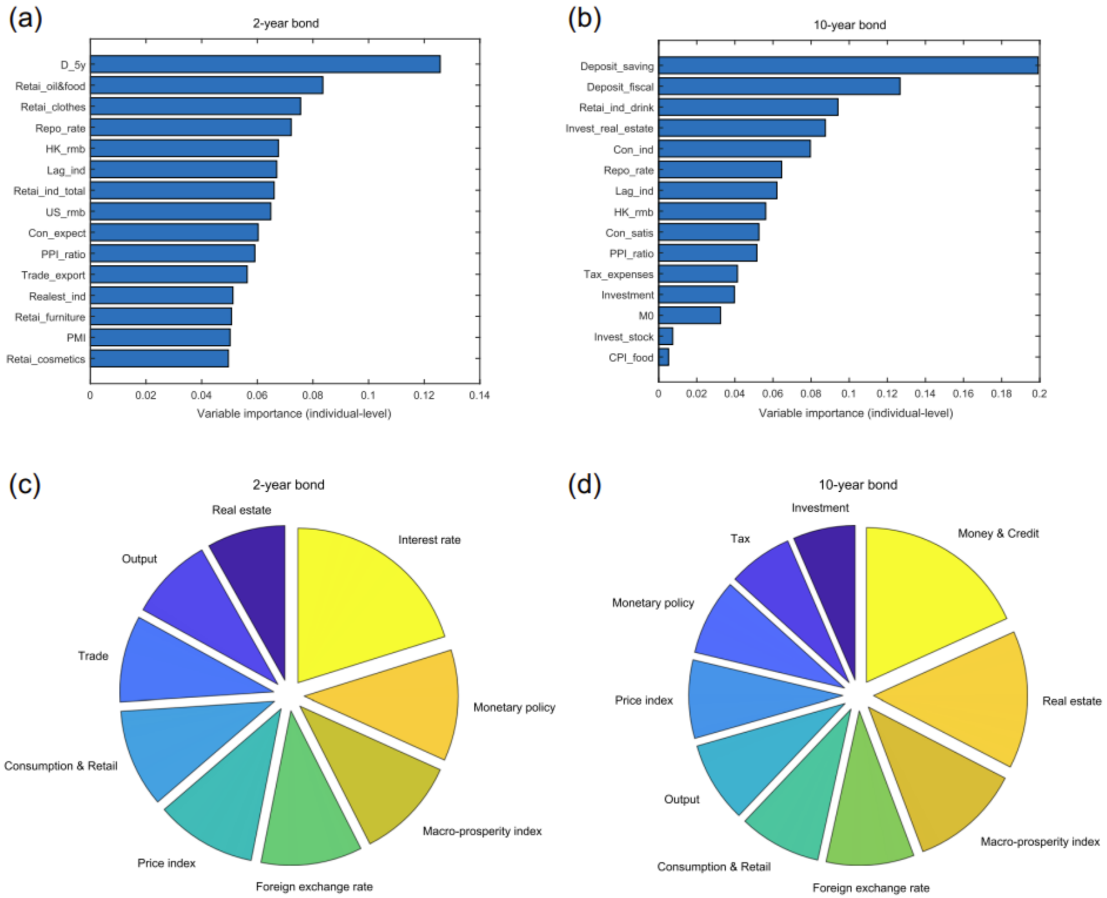
Hybrid-net: Non-linear macro factors + Linear forward rates

4. Economic value of forecasts

Mean-variance utility	$xr_{t+1}^{(2)}$	$xr_{t+1}^{(3)}$	$xr_{t+1}^{(5)}$	$xr_{t+1}^{(7)}$	$xr_{t+1}^{(10)}$	*Multivariate*
Panel A: Forward rates						
Lasso versus EH	0.06***	0.33***	0.29	0.14	−0.03***	0.40
	(0.00)	(0.00)	(0.90)	(0.45)	(0.00)	(0.59)
RF versus EH	−0.06***	0.01***	0.05***	0.17***	0.15***	0.09***
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
NN versus EH	0.18***	−0.03***	0.04	0.06	0.11	0.07*
	(0.00)	(0.00)	(0.18)	(0.71)	(0.43)	(0.05)
Panel B: Forward rates and macroeconomic variables						
Lasso versus EH	−0.09***	0.19***	0.31	−0.10***	−0.03**	−0.09**
	(0.00)	(0.00)	(0.70)	(0.00)	(0.01)	(0.00)
RF versus EH	−0.02***	−0.03***	0.09***	0.18***	0.32***	0.15***
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
NN versus EH	0.08***	0.20***	0.17***	0.33***	0.51*	0.29***
	(0.00)	(0.00)	(0.00)	(0.00)	(0.06)	(0.00)
Panel C: Comparison between forward rates and forward rates plus macroeconomic variables						
Lasso	−0.03***	−0.05***	−0.02***	−0.01**	−0.08***	−0.06***
	(0.00)	(0.00)	(0.00)	(0.02)	(0.00)	(0.00)
RF	0.04**	−0.04***	0.05	0.02***	0.17*	0.05**
	(0.01)	(0.00)	(0.62)	(0.00)	(0.08)	(0.02)
NN	−0.02***	0.20***	0.12***	0.26***	0.34***	0.21***
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)

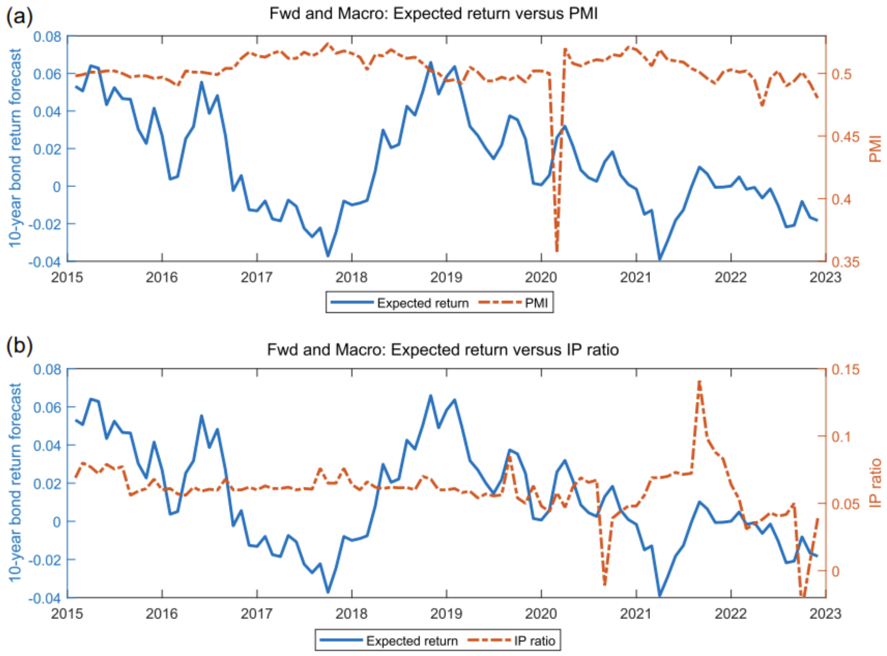
- $\gamma = 5$
- Short selling restrictions ($0 \leq w \leq 1$)

4. Variable importance



- Deposit Rate for the short-term ***(D_5y)***; Savings Deposit for the long-term ***(Deposit_saving)***
- Consistent with the situation in China (high national savings rate and the savings deposit as key driver of economic growth)
- **Importance varies across the term structure**

4. Cyclical pattern of returns



Bond risk premia **should be countercyclical** to economic fundamentals as investors require higher compensation for **recession-related risk**

	Panel A: Correlation		*Panel B: Regression*	
	Forward rates (%)	Forward rates and macro variables (%)	Forward rates	Forward rates and macro variables
IP ratio	−2	−8	−5.39E−04	−0.05
	(0.87)	(0.43)	(0.87)	(0.43)
PMI	−5	−21**	−4.10E−03	−0.32**
	(0.63)	(0.04)	(0.63)	(0.04)
Realized return	5	76***	1.40E−03	0.43***
	(0.66)	(0.00)	(0.66)	(0.00)

Abbreviations: IP, industrial production; NN, neural network; PMI, purchasing managers' index.

Conclusion

- Macroeconomic variables **carry incremental predictive information** for bond risk premia **in addition to yield factors but only when modelled using nonlinear machine learning methods**
- ML methods could provide a more accurate measure of excess bond returns than the OLS regression in China
- **Best implementation: Linear model for the yield portion + nonlinear ML model for the macroeconomic portion of the model**

Appendix: Channel of Predictive Power

- Extract latent factors derived from the NN models and explore if they influence change in the yield curve **(1 - level, 2 - slope, 3 - curvature)**

$$PC_{i,t+1} - PC_{i,t} = b_0 + b_1^T P_t + b_2^T s_t + \varepsilon_{i,t+1}, \quad i = 1, 2, 3,$$

	Level (%)	Slope (%)	Curvature (%)
Lagged three principal components of the yield curve	61	53	92
Neural network (forward rates)	61	53	92
Neural network (forward rates and macroeconomic factors)	76	60	95

Obrigado