International Yield Curves and Currency Puzzles

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Finance Hub seminar

Insper

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Overview

- Objective: fit (and project) bond yields and FX depreciation
- Context: standard (affine) models and the Asset Market View (AMV) fail to predict bond yields and FX depreciation jointly (namely: Backus, Foresi and Telmer, 1997 (BFT))

Literature

AMV fails empirically. Two branches of literature to fix the problem:

- Pursue an incomplete market approach that breaks the equivalence between depreciation rates and the ratio of pricing kernels (e.g., Brandt and Santa-Clara (2002))
- Save the AMV framework by arguing that even if markets are incomplete, one could still obtain depreciation rates via the ratio of pricng kernels as long as those pricing kernels are estimated via variance minimization (e.g., Brennan and Xia (2006), Sarno, Schneider, and Wagner (2012))

Market Completeness and the AMV

Under market completeness and no-arbitrage:

$$E_t\left(M^*_{t,t+1}\cdot R^*_{t+1}\right)=E_t\left(M_{t,t+1}\cdot\frac{S_{t+1}}{S_t}\cdot R^*_{t+1}\right)$$

- R^*_{t+1} is the FC-denominated gross return on an asset
- $M_{t,t+1}$ pricing kernel expressed in USD
- $M^*_{t,t+1}$ pricing kernel expressed in FC

Depreciation rate from AMV is taken from the pricing kernels ratio:

$$\frac{S_{t+1}}{S_t}=\frac{M^*_{t,t+1}}{M_{t,t+1}}$$

Two flaws:

- Volatility anomaly
- Forward-premium puzzle disconnect

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Spanning Regressions

Return on exchange rate:

$$R^{FX}_{t+1} = S_{t+1}/S_t \times 1/Q^{*1}_t$$

Return on domestic n-period bond:

$$R^{n}_{t+1} = Q^{n-1}_{t+1}/Q^{n}_t$$

Return on foreign bond (foreign investor who invest in currency):

$$R^{*,FX}_{t+1} = S_t/S_{t+1} \times 1/Q^{*1}_t$$

Results I

Table I

Spanning Regressions of Currency Returns on Bond and Equity Returns

This table reports the R^2—regular and adjusted—expressed in percent for spanning regressions. We regress annual currency returns of a given country (obtained by investing in a foreign one-period bond) on annual bond returns of maturities $n = 2, 3, \ldots, 10$ years expressed in the same units (USD, denoted \$ returns, or foreign currency, denoted as FC returns). We also combine bond returns with MSCI equity index returns in the last two columns.

		Bond Returns		Bond and Equity Returns	
FX	Type of R^2	\$ Returns Gross Returns	FC Returns	\$ Returns	FC Returns
Euro	R^2	22.41	16.74	24.57	17.09
	R^2_{adj}	20.73	14.93	22.75	15.08
British pound	R^2	17.27	17.30	22.36	17.41
	R^2_{adj}	15.47	15.50	20.47	15.41
Australian dollar	R^2	21.44	24.45	25.84	26.53
	R^2_{adj}	19.50	22.59	23.80	24.52
Japanese yen	R^2	35.03	5.85	35.11	15.44
	R^2_{adj}	33.62	3.58	33.54	13.17
		Log Returns			
Euro	R^2	17.66	16.92	21.10	17.38
	R^2_{adj}	15.87	15.11	19.19	15.38
British pound	R^2	14.52	16.53	22.66	16.71
	R^2_{adj}	12.66	14.71	20.79	14.69
Australian dollar	R^2	23.02	25.25	27.79	26.99
	R^2_{adj}	21.12	23.40	25.81	24.99
Japanese yen	R^2	27.09	5.43	27.38	12.85
	R^2_{adj}	25.50	3.15	25.62	10.50

Results II

Table II

Spanning Regressions of Gross Currency Returns on Bond and Equity Returns in Subsamples

This table reports the R^2—regular and adjusted—expressed in percent for spanning regressions. We regress annual currency returns of a given country (obtained by investing in a foreign one-period bond) on annual bond returns of maturities $n = 2, 3, \ldots, 10$ years expressed in the same units (USD, denoted \$ returns, or foreign currency, denoted as FC returns). We also combine bond returns with MSCI equity index returns in the last two columns.

		Bond Returns		Bond and Equity Returns	
FX	Type of R^2	\$ Returns	FC Returns	\$ Returns	FC Returns
		January 1984 to December 2007			
Euro	R^2	31.59	14.74	31.60	16.94
	R^2_{adj}	29.38	11.98	29.13	13.94
British pound	R^2	13.96	16.75	14.47	20.83
	R^2_{adj}	11.16	14.05	11.37	17.96
Australian dollar	R^2	21.09	24.66	21.95	24.67
	R^2_{adj}	17.99	21.70	18.53	21.37
Japanese yen	R^2	38.89	13.81	38.98	13.81
	R^2_{adj}	36.91	10.52	36.78	10.15
		January 2009 to April 2019			
Euro	R^2	26.91	50.26	39.75	51.84
	R^2_{adj}	21.14	46.33	34.41	47.58
British pound	R^2	29.04	56.88	53.27	56.90
	R^2_{adj}	23.44	53.48	49.14	53.08
Australian dollar	R^2	49.16	47.06	70.01	47.35
	R^2_{adj}	45.14	42.88	67.36	42.69
Japanese yen	R^2	40.68	35.64	40.84	75.62
	R^2_{adj}	36.00	30.56	35.60	73.47

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Model Equations I

VAR(1) of the State:

$$x_t = \mu_x + \Phi_x x_{t-1} + \Sigma_x \varepsilon_t$$

US-short interest rate

$$i_t = \delta_0 + \delta_x^\top x_t$$

SDF in USD:

$$-\log M_{t,t+1} = i_t + \frac{1}{2}\lambda_t^\top \lambda_t + \frac{1}{2}\gamma_t^\top \gamma_t + \lambda_t^\top \varepsilon_{t+1} + \gamma_t^\top \eta_{t+1}$$

Prices of US zero-coupon bonds with maturity n:

$$Q_t^n = E_t\left(M_{t,t+1} \cdot Q_{t+1}^{n-1}\right)$$

Market-prices of risk:

$$\lambda_t = \lambda_0 + \lambda_x x_t$$

Model derived US yields:

$$y_t^n = a_n + b_{n,x^\top}^\top x_t$$

Model Equations II

Depreciation rate (K-dimentional):

$$\Delta s_{t+1} = \mu_s + \Phi_{sx} x_t + \Sigma_{sx} \varepsilon_{t+1} + \Sigma_s \eta_{t+1}$$

Market price of $\eta risk$:

$$\gamma_t = \gamma_0 + \gamma_x x_t$$

Prices of zero-coupon FC bonds with maturity n:

$$Q_t^{*n} = E_t \left(M_{t,t+1} \cdot \frac{S_{t+1}}{S_t} \cdot Q_{t+1}^{*n-1} \right)$$

Model derived foreign yields:

$$y_t^{*n} = a_n^* + b_{n,x}^{*\top} x_t$$

State Variables

$$x_t = \begin{pmatrix} pc_t^{1,\$} \\ pc_t^{2,\$} \\ \Delta_c pc_t^{1,€} \\ \Delta_c pc_t^{2,€} \\ \Delta_c pc_t^{1,£} \\ \Delta_c pc_t^{2,£} \\ \Delta_c pc_t^{1,A\$} \\ \Delta_c pc_t^{2,A\$} \\ \Delta_c pc_t^{1,¥} \\ \Delta_c pc_t^{2,¥} \end{pmatrix} = \begin{pmatrix} \text{first PC from U.S. bonds} \\ \text{second PC from U.S. bonds} \\ \text{first PC from U.S. vs. German bonds} \\ \text{second PC from U.S. vs. German bonds} \\ \text{first PC from U.S. vs. UK bonds} \\ \text{second PC from U.S. vs. UK bonds} \\ \text{first PC from U.S. vs. Australian bonds} \\ \text{second PC from U.S. vs. Australian bonds} \\ \text{first PC from U.S. vs. Japanese bonds} \\ \text{second PC from U.S. vs. Japanese bonds} \end{pmatrix}$$

Estimation

- VAR parameters are exactly identified
- Kalman-filter and MCMC to specify risk-adjusted dynamics of latent states
- Identification restrictions

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Fitted Yields

Table III

Yield Pricing Errors across Countries

This table reports posterior mean estimates of the yield pricing errors in annualized percentage points for the United States, Australia, Euro, Japan, and United Kingdom for both the SFX and UFX model, $100 \times [\mathrm{diag}(\Sigma_y \Sigma_y^\top \times 12)]^{1/2}$. Here, $\Sigma_y \Sigma_y^\top$ is the covariance matrix of the measurement errors for yields of maturity $n = 12, 24, 36, 48, 84, 120$.

	SFX					UFX				
	USA	GER	UK	AU	JP	USA	GER	UK	AU	JP
y_t^{12}	0.06	0.11	0.10	0.11	0.11	0.06	0.11	0.10	0.11	0.11
y_t^{24}	0.06	0.08	0.07	0.06	0.06	0.06	0.08	0.07	0.06	0.06
y_t^{36}	0.05	0.05	0.05	0.04	0.04	0.05	0.05	0.04	0.04	0.04
y_t^{48}	0.04	0.02	0.03	0.02	0.03	0.04	0.02	0.03	0.02	0.03
y_t^{84}	0.03	0.05	0.04	0.04	0.06	0.03	0.05	0.04	0.04	0.06
y_t^{120}	0.08	0.09	0.09	0.07	0.08	0.08	0.09	0.08	0.07	0.07

The FX Disconnect

Table V

Properties of Currency and Interest Rates

In Panel A, we report the sample mean, standard deviation, and autocorrelation of the data. We compare them to the sample moments of model-implied values from the UFX and SFX models. Depreciation rates, short rates, and interest rate differentials are annualized and multiplied by 100. In Panel B, we compare the UIP regression coefficients in the data and in the two models.

Panel A: Summary Statistics

	Mean	SFX	UFX	St.Dev.	SFX	UFX	Autocorr	SFX	UFX
	Δs_t								
Euro	0.899	−16.502	0.899	10.68	38.61	10.68	0.026	0.152	0.026
British pound	−0.595	−19.621	−0.595	10.09	69.00	10.09	0.051	−0.021	0.051
Australian dollar	−0.906	−27.959	0.906	11.49	77.23	11.49	0.054	0.018	0.054
Japanese yen	2.052	−7.674	2.052	10.81	46.95	10.81	0.046	−0.083	0.046
	Short Rates								
USA	3.485	3.686	3.680	0.788	0.868	0.863	0.988	0.992	0.992
Germany	2.659	2.842	2.824	0.770	0.781	0.781	0.984	0.992	0.992
United Kingdom	4.861	5.000	4.969	1.134	1.160	1.156	0.991	0.994	0.994
Australia	6.142	6.266	6.253	1.182	1.195	1.198	0.981	0.986	0.986
Japan	0.952	1.291	1.193	0.685	0.684	0.675	0.963	0.988	0.983
	$y_t^1 - y_t^{*1}$								
Germany	0.722	0.734	0.746	0.670	0.692	0.688	0.984	0.989	0.989
United Kingdom	−1.480	−1.420	−1.399	0.628	0.604	0.605	0.974	0.979	0.979
Australia	−2.935	−2.878	−2.867	0.762	0.746	0.746	0.959	0.968	0.968
Japan	2.429	2.284	2.380	0.614	0.635	0.637	0.965	0.989	0.986

Panel B: UIP Regressions

$$\Delta s_{t+1} = a + b\,(y_t^1 - y_t^{*1}) + \varepsilon_{t+1}$$

	$\widehat{a}$	SFX	UFX	$\widehat{b}$	SFX	UFX
Euro	0.0011	−0.0195	0.0011	−0.260	6.454	−0.270
	(0.0017)	(0.0058)	(0.0017)	(0.973)	(3.688)	(0.945)
British pound	−0.0014	−0.0170	−0.0015	−0.879	−2.582	−0.975
	(0.0015)	(0.0065)	(0.0015)	(1.095)	(5.457)	(1.156)
Australian dollar	−0.0020	−0.0066	−0.0022	−0.693	5.186	−0.770
	(0.0021)	(0.0134)	(0.0020)	(0.684)	(6.066)	(0.703)
Japanese yen	0.0044	−0.0003	0.0043	−1.316	−1.545	−1.277
	(0.0023)	(0.0082)	(0.0023)	(0.907)	(2.895)	(0.856)

Fited FX

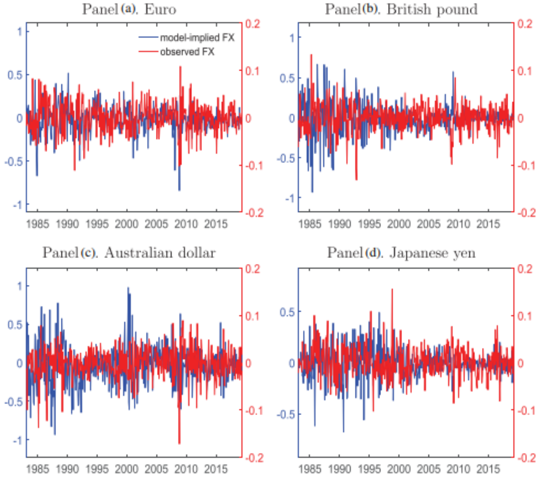

Figure 1. SFX-implied and observed depreciation rates. We plot the depreciation rates implied by the SFX model (blue, vertical axis left) against the observed depreciation rates (red, vertical axis right). (Color figure can be viewed at wileyonlinelibrary.com)

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Variance Decomposition I

Table VI

Variance Decompositions: UFX Model

The table displays how much variation in each variable is attributable to innovations in U.S. yields, foreign yields, and depreciation rates. The numbers are obtained from a VAR-based variance decomposition. We group the yield of the four non-U.S. countries together; similarly, we group depreciation rates.

	U.S. Yields	Foreign Yields	FX
x_{1t}	1	0	0
x_{2t}	1	0	0
x_{3t}	0.70	0.30	0
x_{4t}	0.62	0.38	0
x_{5t}	0.44	0.56	0
x_{6t}	0.28	0.72	0
x_{7t}	0.46	0.54	0
x_{8t}	0.34	0.66	0
x_{9t}	0.43	0.57	0
x_{10t}	0.57	0.43	0
$\Delta s_t^{€}$	0.08	0.02	0.90
$\Delta s_t^{£}$	0.03	0.01	0.96
$\Delta s_t^{A\$}$	0.03	0.01	0.96
$\Delta s_t^{¥}$	0.08	0.03	0.89

Variance Decomposition II

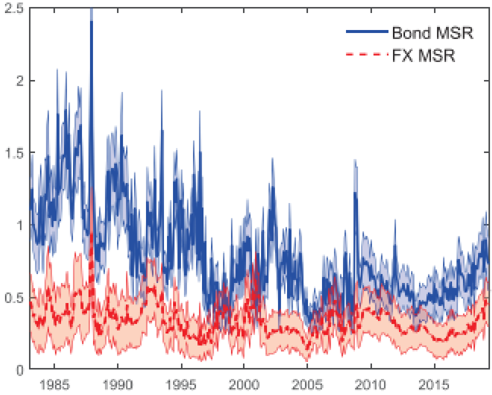

Figure 2. Maximal Sharpe ratios. We decompose conditional volatility of the pricing kernel into the components associated with bond innovations ε, $\sqrt{\lambda_t^\top \lambda_t}$ (blue line, Bond MSR), and depreciation rate innovations η that are unspanned by bonds, $\sqrt{\gamma_t^\top \gamma_t}$ (red line, FX MSR). The corresponding 95% confidence bands are depicted in lighter colors. (Color figure can be viewed at wileyonlinelibrary.com)

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Derived Equations

From FC bond pricing equation and log-normality we can derive the interest rate differential:

$$\Delta_c y_t^{*n} = es_t^n - srp_t^n + vs_t^n$$

News decomposition in t differences in yields, the expected depreciation rate, and the currency risk premium:

$$N_{\Delta y,t}^n \equiv \Delta_c y_t^{*n} - E_{t-1}(\Delta_c y_t^{*n}),$$
$$N_{s,t}^n \equiv es_t^n - E_{t-1}(es_t^n),$$
$$N_{srp,t}^n \equiv srp_t^n - E_{t-1}(srp_t^n).$$

We get the relation:

$$N_{\Delta y,t}^n = N_{s,t}^n - N_{srp,t}^n$$

Which gives us:

$$\text{var}\left(N_{srp,t}^n\right) = \text{var}\left(N_{s,t}^n\right) + \text{var}\left(N_{\Delta y,t}^n\right) - 2\,\text{cov}\left(N_{s,t}^n, N_{\Delta y,t}^n\right)$$

Results (UK)

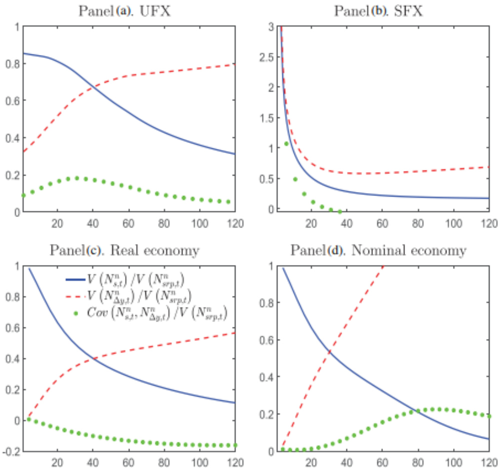

Figure 4. News-based decomposition of currency risk premiums, British pound. We plot the percentage contribution to news about British pound risk premiums, according to $1 = \text{var}(N^n_{s,t})/\text{var}(N^n_{srp,t}) + \text{var}(N^n_{\Delta y,t})/\text{var}(N^n_{srp,t}) - 2\text{cov}(N^n_{s,t}, N^n_{\Delta y,t})/\text{var}(N^n_{srp,t})$, across the different horizons n. We use three models to compute the decomposition: UFX, SFX, and real and nominal versions of the Bansal-Shaliastovich model. (Color figure can be viewed at wileyonlineli-

Thanks!

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