A factor model for option returns

Büchner and Kelly JFE 2022

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Introduction

Introduction

- **Objective:** develop an understanding for the risk return in S&P 500 options using factor pricing approach
- **Method:** instrumented principal components analysis (IPCA)
  - latent factors with time varying loadings
  - well suited to deal with options contracts short lifespan and rapidly changing attributes
- **Findings:**
  1. model outperforms observable factor models in capturing variation in realized options returns
  2. Model with 3 factors is enough to not reject the hypothesis of zero conditional alpha
  3. Trading strategies designed to invest on the efficient frontier of latent factors earn Sharpe ratios as high as 1.8
- Paper is the options market version of Kelly, Pruitt and Su (JFE, 2019): *Characteristics are covariances: A unified model of risk and return*

Overview

Data and Methodology

Data and Methodology: options data

- Options on S&P 500 Index, from January 1996 to December 2017
- Filters:
  - Delta between 0.01 and 0.5 for calls and -0.5 and -0.01 for puts
  - Time-to-maturity from 1 to 12 months
- Focus on **monthly holding period** returns that are delta-hedged daily
- Returns are computed against the underlying mid-price to ensure that returns are well-behaved

Data and Methodology: IPCA

Method developed in Kelly, Pruitt, Su (2019, WP)

Model specification:

$$r_{i,t+1} = \alpha_{i,t} + \beta_{i,t} f_{t+1} + \epsilon_{i,t+1}$$

$$\alpha_{i,t} = z'_{i,t}\Gamma_\alpha + \nu_{\alpha,i,t}, \quad \beta_{i,t} = z'_{i,t}\Gamma_\beta + \nu_{\beta,i,t}$$

- $r_{i,t+1}$ is option i return on month $t+1$
- f_{t+1} is a vector of factors
- $\alpha_{i,t}$ and $\beta_{i,t}$ are time-varying and depend on a vector of options characteristics $z_{i,t}$
- Model is estimated by alternating least squares procedure between f_{t+1} and $\Gamma = [\Gamma_\alpha, \Gamma_\beta]$

Python implementation available on GitHub.

Data and Methodology: Options Characteristics

The following options characteristics are used in the loadings regression:

1. BMS delta
2. time-to-maturity
3. embedded leverage
4. BMS implied vol
5. BMS theta
6. BMS gamma
7. BMS vega

- each variable also has an interaction with a put indicator
- 15 total characteristics after accounting for put interaction and constant
- all characteristics are re-scaled to the range $[-0.5, 0.5]$

Data and Methodology: Characteristic-managed portfolios

Besides studying performance for individual portfolios, paper also examines pricing performance for **characteristic-managed portfolios**:

$$\mathcal{X}_{t+1} = \frac{Z_t' r_{t+1}}{N_{t+1}}$$

Where Z_t is the matrix of characteristics at time t, N_{t+1} is the number of outstanding options at time $t+1$

The managed portfolios are a weighted average of option returns where the weights are determined by the characteristics

Results

Comparison with extant factor models: benchmark models

IPCA (with 1 to 5 factors) performance is compared with 6 benchmark models:

- 5 Observable factors models (allowing for time varying loadings):
  - CAPM
  - FF3: Fama-French three factor
  - FFC4: Carhart four factor model
  - FFCB5: FFC4 + option specific betting-against-beta factor
  - FFCBS6: FFCB5 + Coval and Shumway (2001) straddle factor
- 1 latent factor model:
  - PCA (with 1 to 5 factors)

Comparison with extant factor models: Measuring in-sample performance

Two R^2 statistics measure model performance:

- "total R^2": how well the model describes risk? (realized return variation)

$$\text{Total } R^2 = 1 - \frac{\sum_{i,t}\left(r_{i,t+1} - z'_{i,t}(\hat{\Gamma}_\alpha + \hat{\Gamma}_\beta \hat{f}_{t+1})\right)^2}{\sum_{i,t} r^2_{i,t+1}}.$$

- "predictive R^2": how well the model explains compensation? (difference in average returns)

$$\text{Predictive } R^2 = 1 - \frac{\sum_{i,t}\left(r_{i,t+1} - z'_{i,t}(\hat{\Gamma}_\alpha + \hat{\Gamma}_\beta \hat{\lambda})\right)^2}{\sum_{i,t} r^2_{i,t+1}}.$$

Comparison with extant factor models: R^2

	Panel A: IPCA				
	K				
	1	2	3	4	5
R^2_{tot}	72.28%	79.65%	85.07%	88.90%	90.22%
R^2_{pred}	5.47%	5.54%	6.39%	6.59%	6.77%
$R^2_{tot,x}$	94.41%	96.64%	98.86%	99.39%	99.61%
$R^2_{pred,x}$	7.20%	7.44%	7.90%	7.99%	8.06%
	Panel B: Observable Factors - With Instruments				
	CAPM	FF3	FFC4	FFCB5	FFCBS6
R^2_{tot}	23.81%	25.64%	26.08%	33.15%	49.94%
R^2_{pred}	2.47%	2.38%	2.71%	4.45%	6.24%
$R^2_{tot,x}$	22.55%	25.83%	26.34%	32.68%	56.79%
$R^2_{pred,x}$	3.37%	3.32%	3.58%	5.60%	7.57%
	Panel C: Principal Components Analysis				
	K				
	1	2	3	4	5
R^2_{tot}	18.08%	32.71%	41.70%	47.55%	52.11%
R^2_{pred}	-0.07%	-0.07%	-0.02%	-0.02%	0.15%
$R^2_{tot,x}$	94.09%	97.46%	98.74%	99.44%	99.72%
$R^2_{pred,x}$	7.29%	7.83%	7.87%	7.88%	7.90%

Comparison with extant factor models: Managed Portfolio Alphas

- Another way to assess model pricing performance is with time series regressions of characteristcs portfolios on IPCA Factors and observable factors
- Table show that average IPCA alpha is smaller than FFCBS6

	IPCA		FFCBS6	
	alpha	t-stat	alpha	t-stat
delta	**4.6%**	5.01	**6.9%**	3.69
delta:put	**2.4%**	2.92	2.3%	1.23
ttm	0.4%	0.58	**5.8%**	3.11
ttm:put	1.0%	1.21	**6.0%**	3.25
embed_lev	0.0%	0.02	**3.5%**	2.04
embed_lev:put	0.2%	0.30	**5.1%**	2.74
theta	0.7%	1.21	2.7%	1.91
theta:put	**3.3%**	3.46	**7.9%**	5.36
impvol	-0.9%	-1.27	-3.0%	-1.31
impvol:put	0.5%	0.64	0.3%	0.16
gamma	**-0.6%**	-2.42	-0.4%	-0.27
gamma:put	**2.2%**	2.26	**4.3%**	3.08
vega	0.0%	0.10	-3.4%	-1.91
vega:put	1.0%	1.45	-1.4%	-0.70
const	-0.1%	-0.50	-1.8%	-1.15
Avg. Abs. Alpha	1.2%		3.7%	

Out-of-sample trading strategies

Out-of-sample tangency portfolio, conducted from jan 2001 to dec 2017:

- Recursively estimate IPCA model using only backward looking information
- Based on estimated factors, run portfolio optimization with Ledoit & Wolf covariance estimator
- Portfolios re-leveraged to target 10% ann. volatility

Out-of-sample trading strategies

Results:

	ER	Vol	Sharpe	Skew	Kurtosis	$\alpha(BAB)$	$\alpha(Straddle)$	$\alpha(BAB+Straddle)$
IPCA K=1	0.094	0.096	0.986	-0.977	1.678	0.067 (2.218)	-0.01 (-0.125)	-0.010 (-0.768)
IPCA K=2	0.137	0.091	1.508	-0.598	0.899	0.109 (4.315)	0.041 (1.303)	0.037 (1.545)
IPCA K=3	0.166	0.099	1.673	-0.343	0.587	0.139 (4.920)	0.043 (1.438)	0.040 (1.669)
IPCA K=4	0.179	0.098	1.833	-0.326	0.520	0.151 (6.922)	0.066 (2.530)	0.063 (3.192)
IPCA K=5	0.197	0.109	1.802	-0.783	1.457	0.157 (5.689)	0.070 (1.827)	0.065 (2.310)

- Model with four factors achieves highest sharpe ratio at 1.833
- IPCA models with four and five factors result in significant alphas versus the BAB + Straddle model.

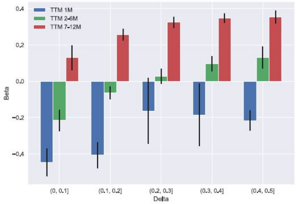
Interpreting the IPCA factors

Trying to recover economic interpretation from the estimated factors (focusing on 3 factors IPCA), the following exercises are conducted:

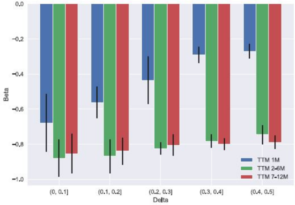
1. Relation to implied volatility surface
2. Relation to observable option return factors
3. Factor behavior across market regimes
4. $\hat{\Gamma}$ analysis

Interpreting the IPCA factors: relation to IV surface

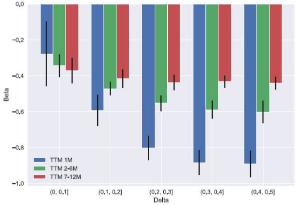
- double sort options in 3 maturity bins and 5 absolute delta bins
- run regression of this portfolios on the 3 IPCA factors
- factor 1 related to vol term structure
- factor 2 and 3 relates to volatility level and skew



(a) Factor 1



(b) Factor 2



(c) Factor 3

Interpreting the IPCA factors: relation to observable factors

Construct 3 portfolios, then regress against IPCA factors:

1. **volatility level:** short options with absolute delta between 0.4 and 0.5
2. **maturity slope:** buy options with maturity between 6 as 12 months, sell with 1 month to maturity
3. **skew:** long calls with delta between 0.1 and 0.2, short puts with delta between -0.2 and -0.1

	F1	F2	F3
Level	-0.10	**0.32**	**1.13**
	(-1.58)	(6.35)	(27.16)
Maturity Slope	**0.71**	**-0.38**	**0.59**
	(10.59)	(-9.44)	(12.82)
Moneyness Skew	**0.53**	**0.58**	**-0.53**
	(8.80)	(7.63)	(-8.61)
R^2_{adj}	72.3%	88.0%	89.3%
No. Obs.	261	261	261
		Shapley-Owen R^2	
Level	9.1%	28.4%	49.4%
Maturity Slope	38.6%	26.2%	20.2%
Moneyness Skew	24.7%	33.4%	19.8%

Interpreting the IPCA factors: relation to observable factors

- observable factors explain close to 90% of the variation in two of our three IPCA factors, more than 70% of in the third factor
- first IPCA factor is most closely related to the maturity slope factor
- the second factor is most closely related to the moneyness skew factor
- third factor is most closely related to the volatility level

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Interpreting the IPCA factors: behavior across regimes

Markets regimes defined following Karakaya (2013):

1. **Price Jump**: S&P returns smaller than -4% past month
2. **Vol Jump**: VIX increase greater than 4%
3. **Severe Bear Mkt**: S&P return smaller than -25% in the previous 12 months
4. **Rising Mkt**: S&P return greater than 25% in the previous 12 months
5. **3 VIX regimes**: constructed from terciles of VIX level

Interpreting the IPCA factors: behavior across regimes table

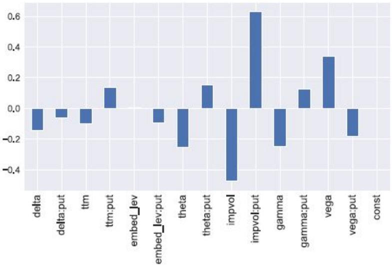
Panel A: IPCA Factors

Period	N. Obs	F1 Mean	F1 StdDev	F1 Sharpe	F2 Mean	F2 StdDev	F2 Sharpe	F3 Mean	F3 StdDev	F3 Sharpe
Full Sample	261	0.06	0.06	**1.04**	0.01	0.04	0.26	0.02	0.02	**1.42**
Price Jump	25	0.14	0.15	**0.93**	-0.11	0.09	**-1.19**	0.01	0.02	0.48
Non Price Jump	236	0.05	0.04	**1.32**	0.02	0.02	**0.98**	0.02	0.01	**1.60**
Vol Jump	50	0.10	0.11	**0.94**	-0.06	0.07	-0.86	-0.01	0.02	-0.52
Non Vol Jump	211	0.05	0.04	**1.33**	0.03	0.02	**1.28**	0.03	0.01	**2.10**
Recession	26	0.06	0.13	0.50	-0.04	0.08	-0.46	0.03	0.02	**1.44**
Non Recession	235	0.06	0.05	**1.32**	0.02	0.03	**0.52**	0.02	0.01	**1.42**
High VIX	65	0.14	0.10	**1.48**	-0.02	0.06	-0.38	0.02	0.02	**1.21**
Medium VIX	130	0.04	0.04	**0.94**	0.02	0.03	**0.85**	0.02	0.01	**1.49**
Low VIX	66	0.02	0.02	**1.22**	0.02	0.01	**1.39**	0.02	0.01	**1.96**

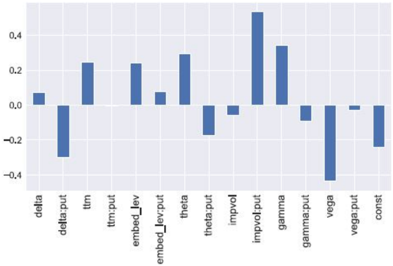
Panel B: Option Level, Slope & Skew

Period	N. Obs.	Level Mean	Level StdDev	Level Sharpe	Maturity Slope Mean	Maturity Slope StdDev	Maturity Slope Sharpe	Moneyness Skew Mean	Moneyness Skew StdDev	Moneyness Skew Sharpe
Full Sample	261	0.02	0.02	**1.03**	0.01	0.01	**0.91**	0.01	0.01	**0.58**
Price Jump	25	-0.03	0.03	-0.82	0.03	0.02	**1.63**	-0.03	0.03	**-0.95**
Non Price Jump	236	0.02	0.01	**1.66**	0.01	0.01	**0.81**	0.01	0.01	**1.29**
Vol Jump	50	-0.03	0.03	-1.09	0.02	0.01	**1.58**	-0.01	0.02	-0.39
Non Vol Jump	211	0.03	0.01	**2.46**	0.01	0.01	**0.70**	0.01	0.01	**1.39**
Recession	26	0.01	0.03	0.32	0.01	0.01	0.98	-0.01	0.02	-0.52
Non Recession	235	0.02	0.01	**1.25**	0.01	0.01	**0.90**	0.01	0.01	**0.98**
High VIX	65	0.01	0.03	0.25	0.02	0.01	**1.63**	0.00	0.02	0.17
Medium VIX	130	0.02	0.01	**1.75**	0.01	0.01	0.62	0.01	0.01	**0.81**
Low VIX	66	0.02	0.01	**2.23**	0.00	0.00	0.53	0.01	0.00	**1.70**

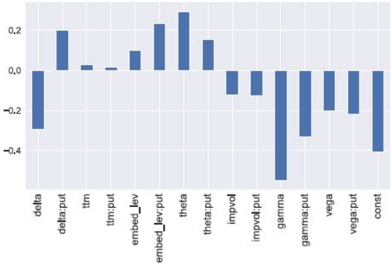
Interpreting the IPCA factors: $\hat{\Gamma}$ analysis



(a) Factor 1



(b) Factor 2



Discussion

Discussion

- Equity factors to price index options
- Level, Maturity and Skew factors (LMS) explains well IPCA factors:
    - IPCA performance vs. LMS?
    - Trading strategy based on LMS efficient frontier?

The End

thanks!