Forest Through the Trees: Building Cross-Sections of Stock Returns

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Roadmap

1. **Introduction**
2. **Methodology**
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Introduction

Motivation

- Asset pricing has two key elements:
  1. A cross-section of test assets (e.g. 25 FF size-value sorted portfolios)
  2. A model for their pricing (e.g. FF 3 factor model)
- Empirical results depend on the choice of the test assets
- Not much is said about how to build them

- This paper: building cross-sections of interpretable test assets that capture the asset pricing information contained in stock characteristics
- And, build a stochastic discount factor (SDF) conditional on a given set of characteristics

The literature of building cross-sections

State of the literature:

- Sorting is usually used to create cross-sections based on 1, 2 or 3 characteristics
- For more than 3 characteristics: stack cross-sections together

Challenges and problems:

- Single sorting ignores any interactions between characteristics
- Multiple sorting is limited to coarse double or triple interactions
- These portfolios do not span the SDF
- Lewellen et al., 2010: strong factor structure makes inference hard

- Sorting captures limited interactions and ignores the conditional distributions between characteristics: does not reflect well the underlying info. and is too low a bar to test models
- Success of simple long-short factors might be due to overly simplistic test assets

Conventional sorting

Double sort on size (LME) and value (BEME)

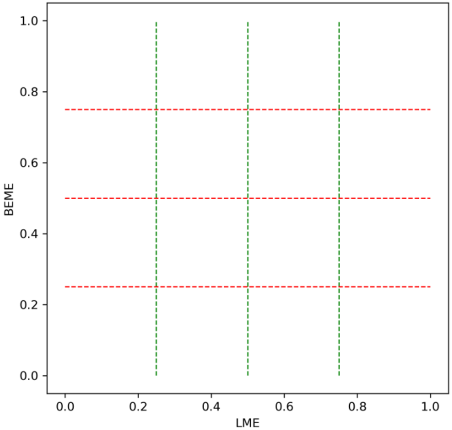

FIGURE 1: Unconditional quantiles, used to build 16 double-sorted portfolios

This paper

Asset Pricing Trees (AP-Trees): cross-section of test assets that reflect interactions and non-linearities between characteristics, based on

- Tree-sorted portfolios: use decision trees to group stocks into managed portfolios based on characteristic interactions
- Pruning the trees: select portfolios that are the most relevant for asset pricing

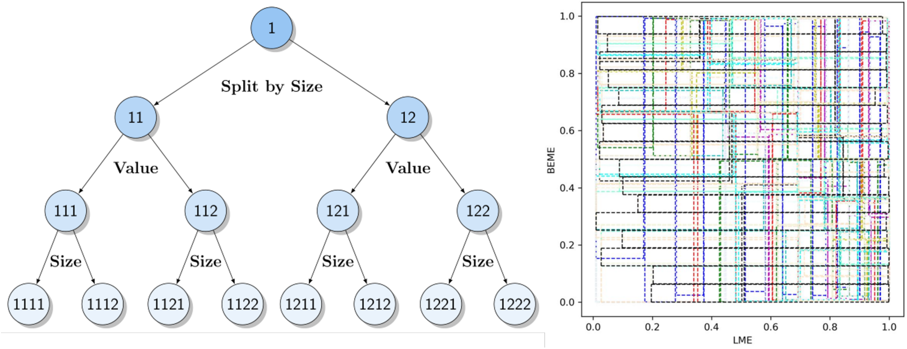

FIGURE 2: Example of a conditional tree based on size and value

This paper

Advantages:

- Portfolios are interpretable and capture arbitrary interactions between characteristics
- Alleviates the curse of dimensionality (not limited to triple sorts)
- Addresses problems of conventional sorts: interactions, dimensionality, repackaging, duplication

Empirically:

- Provides new (and not overstudied) test assets to test asset pricing theories
- AP-Trees have higher OOS Sharpe ratios than sorted portfolios or conventional factors, and are harder to price and more informative
- Test assets that reflect multiple characteristics and their interactions (without stacking)

Decision trees

- Trees can be of depth d based on k characteristics. Each final and intermediate node is a portfolio, and they are conditional on the order of splits

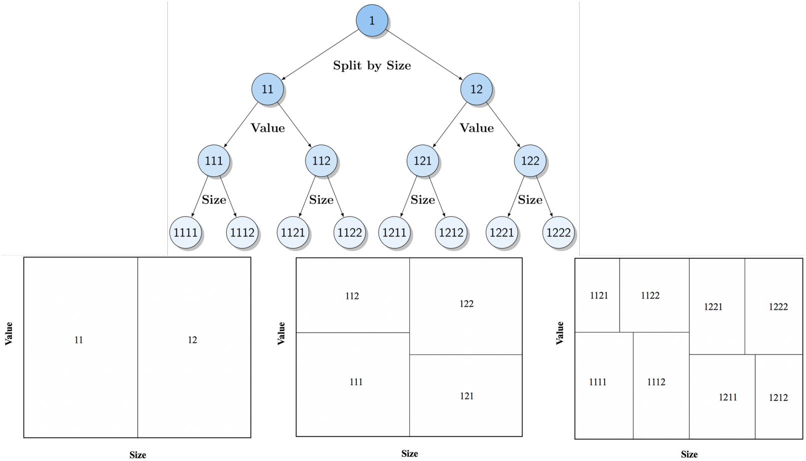

FIGURE 3: Example of a conditional tree based on size and value

Decision trees

- Number of stocks in each portfolio (node) is independent of k, depends only at which level of the tree the portfolio is
  - Root has all stocks
  - First level has each 50% of stocks
  - Terminal nodes will have $\frac{N}{2^d}$ stocks
- Higher level nodes are more diversified, terminal nodes are more granular
- Standard double sort disregards conditional distribution in the characteristics space
- Tree-based portfolios are conditional splits: reflect the joint distribution of characteristics without the need for parametric modeling

Dimensionality reduction in AP-Trees

- Number of possible portfolios grows exponentially on depth and characteristics, pruning is necessary

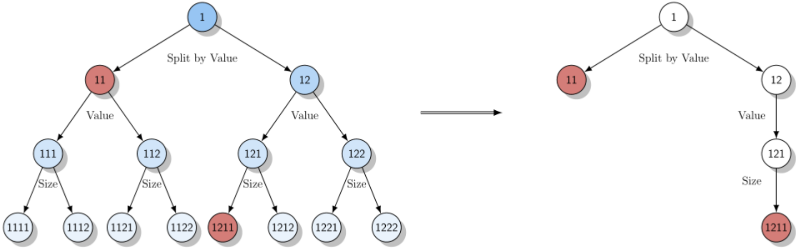

FIGURE 4: Illustration of pruning

- Usual Machine-Learning pruning techniques (bottom-up approach) only work if current split does not affect other nodes
  - Not applicable: decision is local and not Asset Pricing based
- They will select parent nodes, unless the children nodes add asset pricing information

Methodology

Growing AP-Trees

- Choose which characteristics and depth of the tree and group similar stocks together
- Recursively split stocks based on characteristics, creating intermediate and terminal nodes: captures nonlinear dependencies and interactions among characteristics
- Nodes are managed portfolios, and the whole tree is built such that it spans the SDF (i.e. highest OOS Sharpe), aligning with the economic principles that drive asset prices
- Compute the variance and expected return of each managed portfolio, to be used in the pruning stage
- The idea is to create a relatively small number of well-diversified managed portfolios feasible for investors

Pruning AP-Trees and portfolio selection

- Select a small set of AP-Tree nodes with the most non-redundant information for spanning the SDF: *global* problem, so cannot use *local* decision criteria
- Solve a mean-variance optimization with Elastic-Net problem applied to all nodes of all trees to select optimal portfolios
- Elastic-Net approach (Ridge + Lasso) ensures a balance between bias and variance in the portfolio weights, reducing the risk of overfitting: shrinkage + regularization
- Develop an algorithm for selecting the robust portfolio weights from AP-Trees
- Tuning parameters (target mean μ_0, Lasso weight λ_1 and Ridge weight λ_2) are treated as fixed in the prunning stage and chosen separately on the validation data

Pruning AP-Trees and portfolio selection algorithm

Divide original sample of data into three non-intersecting subsamples for use in training, validation, and testing of the model. Grow the AP-Trees on the training set, get $\hat{\Sigma}$ and $\hat{\mu}$, then

1. For a given set of values of tuning parameters μ_0, λ_1 and λ_2 , use *training* data to solve

$$\begin{aligned} \min \quad & \frac{1}{2} w^\top \hat{\Sigma} w + \lambda_1 \|w\|_1 + \frac{1}{2}\lambda_2 \|w\|_2^2, \\ \text{subject to} \quad & w^\top \mathbb{1} = 1 \\ & w^\top \hat{\mu} \geq \mu_0, \end{aligned} \tag{1}$$

2. Use *validation* data to estimate the Sharpe ratio of the optimal portfolio from eq. (1) as a function of tuning parameters. Select parameters that maximize Sharpe ratios, store resulting portfolio selection and their weights in the SDF
3. Use *testing* data to trace the fully out-of-sample performance of the SDF and the properties of the individual portfolios in the chosen cross-section

Pruning AP-Trees and portfolio selection results

The approach leads to three crucial features:

1. It shrinks the contribution of the assets that do not help span the SDF
2. It shrinks the sample average portfolio returns toward their cross-sectional average value, since the highest/lowest sample returns in a large cross-section are likely to be over/under-estimated
3. It includes a lasso-type regularization and ridge-type shrinkage to obtain a sparse representation of the SDF, selecting a small number of AP-Tree basis assets

Empirical Design

Data

- Monthly stock returns from CRSP merged with Compustat
- Data sample: January 1964 to December 2016 (53 years)
- 10 characteristics built: Size, Value, Operating Profitability, Investment, Momentum, Short-term Reversal, Long-term Reversal, Accrual, Asset Turnover, and Idiosyncratic Volatility
  - Portfolios available in K. French website
- Basic set of characteristics to build trees and triple sorts: Size + 2 other: **36 cross-sections**

Empirical implementation

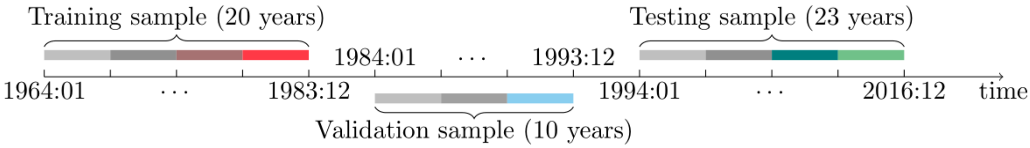

FIGURE 5: Timeline of the empirical strategy

Training: build constant portfolio weights (value-weighted):

- Estimate AP-Trees with 40 basis assets from penalized portfolio frontier applied to all tree-based portfolios. Trees are of depth four and based on three characteristics (Size + 2 other)
- Build portfolios based on triple sorts and other benchmark test assets
- Construct weights of SDF based on AP-Tree and triple sorts

Validation: select tuning parameters for elastic net for efficient frontier (formed on the training sample). Select the parameters that maximize the Sharpe ratio

Test: Compare the performance out-of-sample of different basis assets

Asset pricing performance

Asset pricing test portfolios:

- AP-Tree with 40 basis assets (similar results for AP-Tree with 10 basis assets)
- 32 TS: triple sorted with 32 basis assets (only one size split)
- 64 TS: triple sorted with 64 basis assets (two size splits)

Factor models:

- Fama-French three- and five-factor models
- XSF: A cross-section-specific model: market + three long-short portfolios of charcts. used
- FF11: An 11-factor model, market factor + 10 long-short portfolios of all 10 characteristics

For each of the 36 cross-sections:

- OOS Sharpe ratio
- α (Pricing error): t-stat of OOS time-series pricing error of SDF
- $XS-R^2$ (Cross sectional R^2): relative magnitude of pricing errors in a given cross-section, i.e., $XS-R^2 = 1 - \frac{N}{N-K}\frac{\sum_{i=1}^{N}\alpha_i^2}{\sum_{i=1}^{N}E\left[R_i^{ex}\right]^2}$

Sharpe ratios

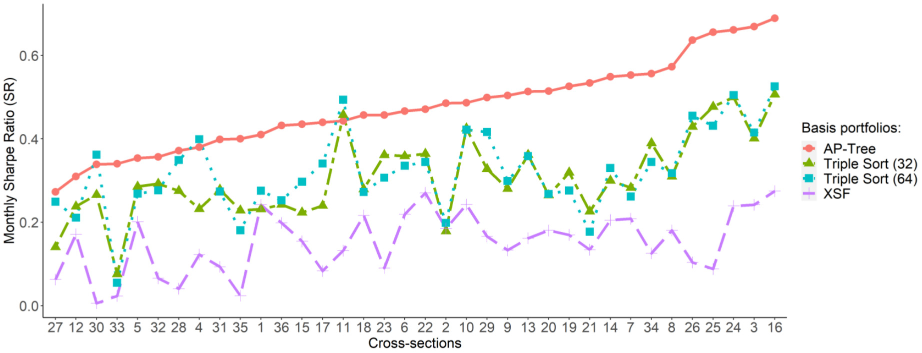

FIGURE 6: Monthly out-of-sample Sharpe ratios

- AP-Trees have OOS SR up to three times higher: reflect more underlying pricing info
- XSF have about half of the SR of TS: standard long-short factors miss important cross-sectional information, not accounting for any characteristic interaction

SDF unspanned by factors: FF5

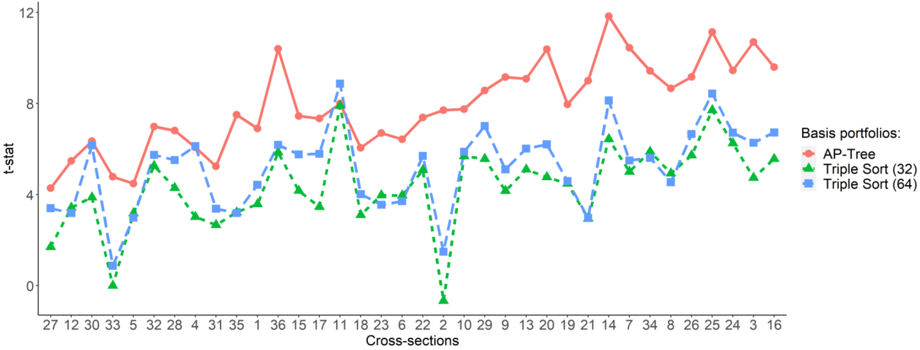

FIGURE 7: SDF α: t-statistics of α relative to Fama-French 5 factor model

- Pricing error of SDF built from AP-Trees is the highest
- FF5 successfully spans some of the TS cross-sections, but fails to do so in AP-Trees
- XS 2 (size, value, and profitability): AP-Tree α t-stat is 8, for TS it is ≈ 0

Relative magnitude of pricing errors

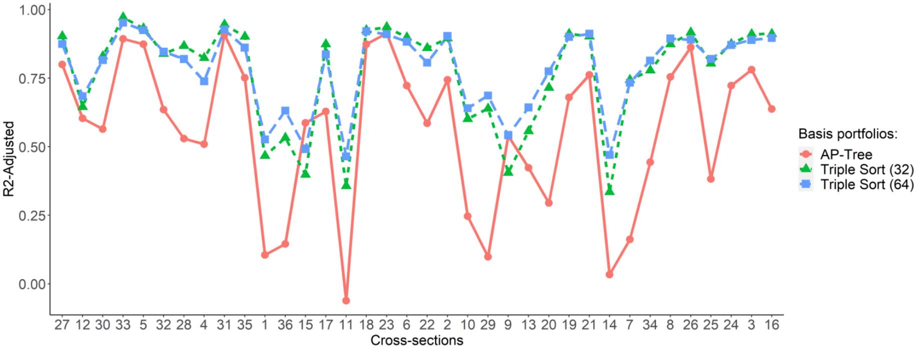

FIGURE 8: Cross-sectional R^2 with AP-Trees and triple sorts as basis assets w.r.t. FF5

- Conventional factor models also fail to explain the spread of average returns within AP-Tree cross-section
- Typically above 80% is considered good, but for AP-Trees it is on avg below that

Nonlinearities and characteristic interactions

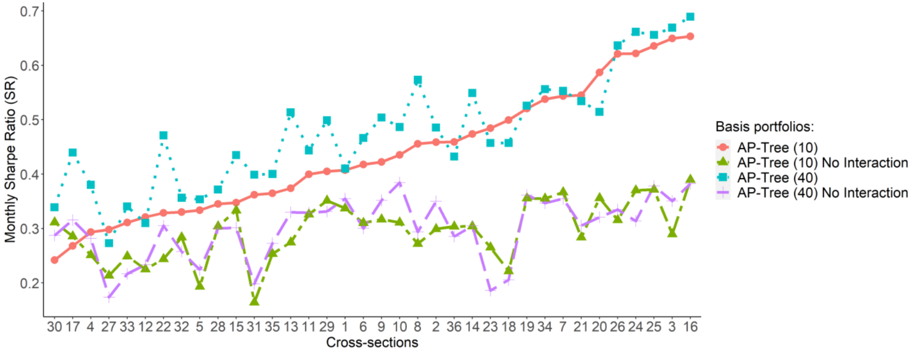

FIGURE 9: AP-Trees: the impact of interaction nodes

- Remove all interaction nodes, essentially reflecting the universe of single-sorted test assets
- OOS Sharpe ration without interactions are half of the benchmark AP-Trees
- Source of outperformance of AP-Trees

Zooming into a cross-section

		Type of cross-section			
		AP-Trees (10)	AP-Trees (40)	TS (32)	TS(64)
SDF SR		0.65	0.69	0.51	0.53
α	FF3	0.94	0.90	0.75	0.84
		[10.11]	[11.03]	[7.40]	[8.13]
	FF5	0.81	0.76	0.47	0.61
		[8.76]	[9.60]	[5.57]	[6.73]
	XSF	0.81	0.76	0.46	0.61
		[8.77]	[9.46]	[5.39]	[6.69]
	FF11	0.89	0.80	0.37	0.65
		[9.12]	[9.60]	[4.29]	[6.91]
$XS\text{-}R^2$	FF3	18.0%	51.0%	82.0%	82.0%
	FF5	11.0%	64.0%	91.0%	90.0%
	XSF	28.0%	65.0%	91.0%	90.0%
	FF11	–	42.0%	92.0%	87.0%

TABLE 1: Cross-sections based on size, operating profitability, and investment

- AP-Trees cross-sections have larger SR's and α's than triple sorted portfolios, and lower $XS - R^2$: AP-Tree is more informative and harder to price
- Performance using 10 portfolios close to that of 40 portfolios

Zooming into a cross-section

Portfolio ID	Portfolio description	% of stocks	WV size quantile	α_{FF3}	α_{FF5}	α_{XSF}	$\alpha_{FF_{11}}$
Panel A: 10 AP-Tree portfolios							
1 (1111.1)	market	100%	0.95	–	–	–	–
2 (1111.121)	medium caps	25%	0.71	-0.03	-0.37	-0.05	-0.03
				[-0.34]	[-0.36]	[-0.40]	[-0.26]
3 (3111.11)	low profitability	50%	0.94	0.03	-0.11**	-0.11**	-0.11**
				[-0.54]	[-2.33]	[-2.20]	[-2.46]
4 (3111.11111)	low investment, small caps	6.25%	0.07	1.49***	1.32***	1.32***	1.63***
				[6.14]	[5.33]	[5.33]	[6.47]
5 (3211.1121)	low investment, high profit., small caps	12.5%	0.51	0.23***	0.09	0.07	0.06
				[2.95]	[1.19]	[0.66]	[0.83]
6 (1221.1111)	small caps, low profitability	12.5%	0.37	-0.37***	-0.34***	-0.34***	-0.29**
				[-2.81]	[-3.00]	[-3.01]	[-2.41]
7 (2221.11111)	low profitability, small caps	6.25%	0.19	-0.16	-0.22	-0.21	-0.05
				[-0.717]	[-0.95]	[-0.89]	[0.21]
8 (3331.12221)	high investment, small caps	6.25%	0.45	-0.82***	-0.62***	-0.62***	-0.46***
				[-5.86]	[-4.81]	[-4.78]	[-3.75]
9 (2122.11111)	low profitability, small caps	6.25%	0.27	-0.37*	-0.43**	-0.42**	-0.20
				[-1.95]	[-2.21]	[-2.13]	[-1.00]
10 (3133.12122)	high investment, small caps	6.25%	0.48	-0.84***	-0.64***	-0.64***	-0.50***
				[-5.92]	[-4.84]	[-4.81]	[-3.92]

TABLE 2: 10 AP-Trees portfolios

- Only 5 portfolios are final nodes of the trees, others include many more assets
- α's of individual portfolios are mostly different than 0, and they are not from extreme quantiles of the characteristics, but are characterized by a complex interaction
- Contrasts triple sorted portfolios which are extremely granular and are composed of very few stocks

AP-Trees in large dimension

- Stacking small-cross sections has been the only way to consider many more characteristics, which repackages the same assets into different portfolios with no evidence that it reflects different economic risks No universal benchmark for stacking portfolios, they consider:
  - Sets of 10 quintile ports., uniformly sorted by characteristic (50 assets altogether)
  - Sets of 10 decile-sorted ports (100 assets)
  - A combination of six double-sorted ports., each based on size and some other charac. (54 assets)
  - A combination of 25 double-sorted portfolios, with each based on size and some other charac. (225 assets)
- For tree-based portfolios, they combine selected assets (10/40 portfolios) from each of the 36 cross-sections

AP-Trees in large dimension

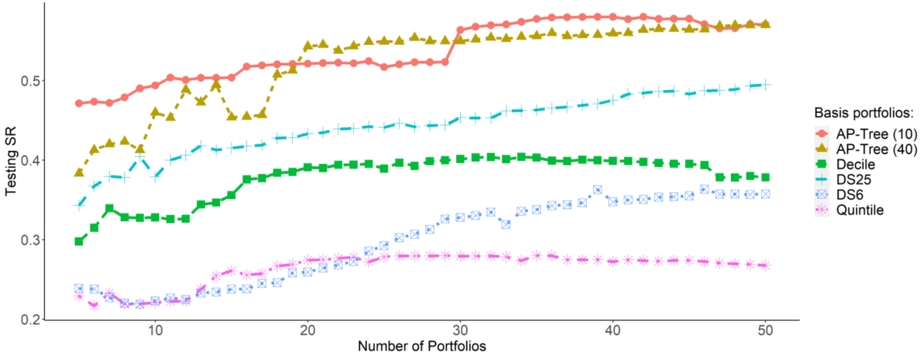

FIGURE 10: Sharpe ratios of the SDFs spanned by AP-Trees and traditional sorts

- Quintile sorted portfolios are the least informative
- Combination of 25 double sorted portfolios is the best within the standard framework
- AP-Trees are clearly superior than any of them, allowing for interactions between characteristics

Evaluating asset pricing models

Statistics	Porfolios	Carhart	FF-3	FF-5	FF-6	Hirshleifer, Jiang (2010)	Q5	Stambaugh, Yuan (2017)
Avg Abs Alpha	Decile 100	0.0014	0.0018	0.0013	**0.0011**	0.0012	0.0012	0.0013
	DS 225	0.0025	0.0023	**0.0021**	0.0026	0.0023	0.0031	0.0032
	DS 40	0.0034	0.0039	0.0032	0.0031	**0.0029**	0.0037	0.0040
	DS 70	0.0032	0.0034	**0.0029**	0.0031	0.0028	0.0035	0.0039
	AP-Trees 40	0.0071	0.0081	0.0067	**0.0063**	0.0065	0.0064	0.0067
	AP-Trees 70	0.0058	0.0065	0.0053	**0.0052**	0.0053	0.0054	0.0057
# Signif. Alphas (%)	Decile 100	10 (10%)	18 (18%)	10 (10%)	8 (8%)	**5 (5%)**	6 (6%)	9 (9%)
	DS 225	77 (34%)	58 (26%)	**39 (17%)**	68 (30%)	47 (21%)	75 (33%)	74 (33%)
	DS 40	14 (35%)	16 (40%)	8 (20%)	12 (30%)	**7 (18%)**	11 (28%)	13 (33%)
	DS 70	28 (40%)	25 (36%)	**16 (23%)**	25 (36%)	17 (24%)	25 (36%)	27 (39%)
	AP-Trees 40	23 (58%)	29 (73%)	20 (50%)	18 (45%)	18 (45%)	**14 (35%)**	**14 (35%)**
	AP-Trees 70	37 (53%)	39 (56%)	27 (39%)	28 (40%)	28 (40%)	**24 (34%)**	**24 (34%)**
XS-R^2	Decile 100	0.88	0.78	0.89	**0.92**	0.91	0.91	0.89
	DS 225	0.82	0.83	**0.84**	0.77	**0.84**	0.70	0.67
	DS 40	0.65	0.54	0.66	0.56	**0.69**	0.44	0.37
	DS 70	0.72	0.69	0.74	0.65	**0.75**	0.57	0.52
	AP-Trees 40	0.17	0.12	**0.23**	0.03	0.20	-0.02	-0.12
	AP-Trees 70	0.36	0.33	**0.40**	0.24	0.38	0.18	0.09
SDF α (t-stat)	Decile 100	6.47	7.03	5.93	5.64	5.60	5.03	**4.95**
	DS 40	8.64	9.05	7.95	7.82	7.52	6.64	**6.62**
	DS 70	9.04	9.48	8.36	8.19	7.95	**6.81**	6.96
	AP-Trees 40	9.78	10.08	8.95	8.98	8.35	7.86	**7.33**
	AP-Trees 70	10.92	11.32	10.46	10.34	9.57	8.92	**8.50**
SR out-of-sample		0.20	0.16	0.21	0.23	0.32	**0.43**	0.30

Table 3: Sharpe ratios of the SDFs spanned by AP-Trees and traditional sorts

- 100-decile sorted portfolios are easily priced by models, double-sorted too, but to a lesser extent. AP-Trees are considerably harder to price
- α's are misleading: ignores portfolio repackaging and can focus on redundant test assets

Conclusion

- They proposed a new way to build characteristic-based cross-sections of returns with decision trees that:
  1. Capture the complex interactions and nonlinearities between characteristics
  2. Provide test assets that are closer to spanning the SDF and are harder to price than conventional cross-sections
  3. Act as the the building block for an SDF that performs well out-of-sample

- They show that conventional cross-sections (and stacking of them) do not fully capture the information contained in the characteristics, and are a bad description of expected returns

Thank you!

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References I

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